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OPTIMIZATION OF PARAMETRIC CONSTANTS FOR CREEP-RUPTURE DATA BY MEANS OF LEAST SQUARES

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OPTIMIZATION OF PARAMETRIC CONSTANTS FOR CREEP-RUPTURE DATA BY MEANS OF LEAST SQUARES

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SUMMARY

An objective method utilizing least squares is presented for the determination of the optimum parametric constants for stress-rupture data. The method is applied to both isostress and isothermal data for the parameters proposed by Larson and Miller, Manson and Haferd, and by Dorn. Several examples are treated in detail, and it was found that the method gives good results. It is shown that the values of the constants for the parameter proposed by Manson and Haferd are not critical as long as T_a and $\log t_a$ appear in the proper combination. In addition to optimization, the chief utility of the method lies in the fact that it gives the same results for a given set of data no matter who makes the analysis, which is not the case for the graphical methods presently employed.

INTRODUCTION

The problem of extrapolating and correlating high-temperature creep- and stress-rupture data is presently receiving considerable attention. In particular, the ability to extrapolate short-time rupture data can greatly reduce costly experimental programs and also reduce time delays in choosing a suitable material for a given application.

The most widely used techniques at present for extrapolating stress-rupture data are the so-called parameter methods. These methods assume that by plotting the creep-rupture data for a given material in an appropriate parametric form a single master curve results which can then be used for interpolation and extrapolation purposes. The three best known parameter methods are those of Larson and Miller (ref. 1), Manson and Haferd (linear parameter) (ref. 2), and Dorn (ref. 3).

The Larson-Miller method assumes that a plot of log rupture time against the reciprocal of the absolute temperature at a given constant nominal stress is linear. Furthermore, it is assumed that all such constant-nominal-stress lines intersect at a common point (0,-C) (fig. 1(a)). A plot, therefore, of the stress σ against $(\log t + C)(T + 460)$ should produce a single master curve valid for all stresses and temperatures as shown in figure 1(b). Originally, the constant C was taken equal to 20; however, it is now generally recognized that for best results it will vary with the material.

The linear parameter method assumes that a plot of log rupture time against temperature in degrees Fahrenheit at a given constant nominal stress is linear and that all such lines converge to a common point $(T_a, \log t_a)$ as shown in figure 2(a). The constants T_a and $\log t_a$ are determined from the data for a given material in a given condition. A plot, therefore, of the stress σ against $\frac{T - T_a}{\log t - \log t_a}$ should produce a master curve, as shown in figure 2(b), valid for all stresses and temperatures.

The Dorn method assumes that a plot of log rupture time against the reciprocal of the absolute temperature at a given constant nominal stress is linear and that all such straight lines are parallel with slope D as shown in figure 3(a). A plot, therefore, of the stress against $te^{-\frac{2.3\,D}{T+460}}$ should produce a single master curve valid for all stresses and temperatures (fig. 3(b)).

It is seen from the foregoing discussion that, in order to make use of these parameter methods, certain material constants must be determined. To do this, the general practice has been to plot creep-rupture data for a given material as shown in figure 1(a), 2(a), or 3(a), depending on the selected parameter. The desired constants can then be obtained visually by appropriate extrapolation. Only when data are given in constant-nominal-stress form, can these plots be obtained directly. In the more usual case where the data are isothermal, several cross plots must first be made before figures such as 1(a), 2(a), and 3(a) can be constructed.

Since creep-rupture data generally have appreciable scatter, it is apparent that the results of such visual cross-plotting will depend upon the judgment of the individual analyzing the data. Furthermore, some experience in the field of material evaluation is necessary in constructing the various plots.

This report presents an analytical method for determining the best values of the constants for the three parameters discussed. The method which is based on the standard least-squares procedure makes use of the original raw data and requires no judgment on the part of the analyzer. Moreover, since it is a least-squares procedure, it gives the statistically most probable values of the desired constants. The method is first presented for the case of constant-nominal-stress data and then for the more usual case of isothermal data. Examples are presented for several materials for each of the two cases.

It is not the object of this report to discuss the merits of the various parameters used. This has been discussed at length in references 4 and 5 where it is indicated that in general the linear parameter gives better agreement with experiment for extrapolated stress-rupture times than either the Larson-Miller or Dorn parameters.

ANALYSIS

Constant-Nominal-Stress Data

Larson-Miller parameter. - Consider a set of constant-stress data as shown in figure 1(a). On the basis of the Larson-Miller parameter, it is assumed that a set of straight lines intersecting the $\log t$ axis at $\log t$ equal to $-C$ can be fitted to these data. The equation of the straight line passing through the data for the first constant-stress line can be written as

$$y^{(1)} = -C + b_1 \tau^{(1)} \tag{1a}$$

and for the second isostress line,

$$y^{(2)} = -C + b_2 \tau^{(2)} \tag{1b}$$

Thus, for any isostress line, say the j^{th},

$$y^{(j)} = -C + b_j \tau^{(j)} \tag{1c}$$

where

$y \equiv \log t$

$b_j \equiv$ slope of j^{th} isostress line

$\tau \equiv \dfrac{1}{T + 460}$

and the superscripts designate the particular constant-stress line under consideration.

To find the best set of lines fitting the data and intersecting at the point $-C$, the sum of the squares of the deviations S of the actual data points from the lines (the residuals) is minimized. Thus,

$$S = \sum_{i=1}^{n_1} \left[y_i^{(1)} + C - b_1\tau_i^{(1)}\right]^2 + \sum_{i=1}^{n_2} \left[y_i^{(2)} + C - b_2\tau_i^{(2)}\right]^2 + \ldots +$$

$$\sum_{i=1}^{n_p} \left[y_i^{(p)} + C - b_p\tau_i^{(p)}\right]^2 = \text{minimum} \tag{2}$$

where p is the number of isostress lines and n_1, n_2, and so forth are the number of data points for each line.

In order to find the values of C and the b_j that will make S a minimum, S is differentiated with respect to C and the b_j, and the resulting equations are set equal to zero. This results in

$$\left.\begin{array}{l}
-nC + b_1\sum_{i=1}^{n_1}\tau_i^{(1)} + b_2\sum_{i=1}^{n_2}\tau_i^{(2)} + \ldots + b_p\sum_{i=1}^{n_p}\tau_i^{(p)} = \sum_{i=1}^{n} y_i \\
\text{and} \\
\qquad -A_1C + b_1B_1 = C_1 \\
\qquad -A_2C + b_2B_2 = C_2 \\
\qquad \quad \vdots \qquad \quad \vdots \qquad \quad \vdots \\
\qquad -A_pC + b_pB_p = C_p
\end{array}\right\} \tag{3}$$

where

$$\left.\begin{aligned} A_j &= \sum_{i=1}^{n_j} \tau_i^{(j)} \\ B_j &= \sum_{i=1}^{n_j} \left[\tau_i^{(j)}\right]^2 \\ C_j &= \sum_{i=1}^{n_j} y_i^{(j)} \tau_i^{(j)} \end{aligned}\right\} \tag{4}$$

and n is the total number of data points:

$$n = n_1 + n_2 + \ldots + n_p$$

Solution of equations (3) for C and b_j gives

$$\left.\begin{aligned} C &= -\frac{\displaystyle\sum_{i=1}^{n} y_i - \sum_{j=1}^{p} \frac{A_j C_j}{B_j}}{\displaystyle n - \sum_{j=1}^{p} \frac{A_j^2}{B_j}} \\ b_j &= \frac{C_j + A_j C}{B_j} \end{aligned}\right\} \tag{5}$$

By means of equations (5), the best value of the Larson-Miller parameter C can be directly computed for a given set of constant-stress data. The best lines intersecting at $-C$ can also be plotted by using the second of equations (5).

Dorn parameter. - To determine the best slope D of a set of parallel constant-stress lines as shown in figure 3(a), a similar procedure as for the Larson-Miller parameter is used. The equation of any one of the isostress lines can be written as follows:

$$y^{(j)} = a_j + D\tau^{(j)} \tag{6}$$

The expression to be minimized now becomes

$$S = \sum_{i=1}^{n_1} \left[y_i^{(1)} - a_i - D\tau_i^{(1)}\right]^2 + \sum_{i=1}^{n_2} \left[y_i^{(2)} - a_2 - D\tau_i^{(2)}\right]^2 + \ . \ . \ . \ +$$

$$\sum_{i=1}^{n_p} \left[y_i^{(p)} - a_p - D\tau_i^{(p)}\right]^2 = \text{minimum} \tag{7}$$

Differentiating with respect to the a_j and D and setting the resulting expression equal to zero give

$$\left.\begin{aligned} n_j a_j + A_j D &= D_j \\ a_j \sum_{i=1}^{p} A_i + D \sum_{i=1}^{p} B_i &= \sum_{i=1}^{p} C_i \end{aligned}\right\} \tag{8}$$

where

A_i, B_i, and C_i are as defined in equations (4) and

$$D_j = \sum_{i=1}^{n_j} y_i^{(j)} \tag{8a}$$

The solution of equations (8) gives

$$\left.\begin{aligned} D &= \frac{\sum_{j=1}^{p} C_j - \frac{A_j D_j}{n_j}}{\sum_{j=1}^{p} B_j - \frac{A_j^2}{n_j}} \\ a_j &= \frac{D_j - DA_j}{n_j} \end{aligned}\right\} \tag{9}$$

The Dorn parameter D can, therefore, be calculated directly from equations (9).

Linear parameter. - A similar approach as in the preceding can be used for determining the constants T_a and $\log t_a$ (fig. 2(a)) for the linear parameter. The equation for any of the isostress lines is

$$y^{(j)} = y_a - b_j T_a + b_j T^{(j)} \tag{10}$$

where

$$y_a \equiv \log t_a$$

Equation (10), however, is nonlinear in the unknown constants because of the term $b_j T_a$. Minimizing the sum of the squares of the deviations would, therefore, lead to a set of nonlinear algebraic equations which would be very difficult to solve. To avoid this difficulty, two alternate approaches have been used. In the first approach, a value is assumed for T_a. Equation (10) is then written as follows:

$$y^{(j)} = y_a + b_j \left[T^{(j)} - T_a\right] \tag{10a}$$

Equation (10a) has exactly the same form as equations (1) and the solution is, therefore, given by equations (5) with C replaced by $-y_a$ and τ replaced by $T - T_a$. Once the best values of y_a and b_j are found for the assumed value of T_a, the sum S is computed by equation (2) (again replacing C by $-y_a$ and τ by $T - T_a$). A new value of

T_a is chosen, and the calculation is repeated giving a new value of S. The value of T_a for which S is a minimum is the correct value. Since the results for the linear parameter are generally insensitive to the exact value of T_a as long as the corresponding value of y_a is used, the previous trial-and-error procedure need, in general, be carried out only a few times in order to obtain a satisfactory value for T_a.

An alternate simpler approximate method which does not involve trial and error can also be used. In this method, the nonlinear term b_jT_a is temporarily grouped with y_a, and equation (10) is written as follows:

$$y^{(j)} = d_j + b_jT^{(j)} \tag{11}$$

where

$$d_j \equiv y_a - b_jT_a$$

Equation (11) is now linear in the unknown constants, d_j and b_j, and these can be found by least squares as before. Thus, the sum S to be minimized now becomes

$$S = \sum_{i=1}^{n_1} \left[y_i^{(1)} - d_1 - b_1T_i^{(1)}\right]^2 + \sum_{i=1}^{n_2} \left[y_i^{(2)} - d_2 - b_2T_i^{(2)}\right]^2 + \cdots +$$

$$\sum_{i=1}^{n_p}\left[y_i^{(p)} - d_p - b_pT_i^{(p)}\right]^2 = \text{minimum} \tag{12}$$

Differentiating with respect to the d_j and b_j leads to

$$\left.\begin{aligned} n_jd_j + A_jb_j &= D_j \\ A_jd_j + B_jb_j &= C_j \end{aligned}\right\} \tag{13}$$

where A_j, B_j, C_j, and D_j are as previously defined and

$$\tau_i \equiv T_i$$

Solving equations (13) gives

$$\left.\begin{aligned} d_j &= \frac{B_jD_j - A_jC_j}{n_jB_j - A_j^2} \\ b_j &= \frac{n_jC_j - A_jD_j}{n_jB_j - A_j^2} \end{aligned}\right\} \qquad (14)$$

Since the best values for d_j and b_j have been determined from equations (14), the best values of T_a and y_a can now be found as follows:

$$d_j = y_a - b_jT_a$$

and it is desired to find the best values for y_a and T_a. The following sum is, therefore, minimized:

$$\sum_{j=1}^{p} (d_j - y_a + b_jT_a)^2 = \text{minimum} \qquad (15)$$

which gives

$$\left.\begin{aligned} py_a - T_a \sum_{j=1}^{p} b_j &= \sum_{j=1}^{p} d_j \\ y_a \sum_{j=1}^{p} b_j - T_a \sum_{j=1}^{p} (b_j)^2 &= \sum_{j=1}^{p} d_jb_j \end{aligned}\right\} \qquad (16)$$

Therefore,

$$\left.\begin{aligned} T_a &= \frac{\sum_{j=1}^{p} d_j \sum_{j=1}^{p} b_j - p \sum_{j=1}^{p} b_jd_j}{p \sum_{j=1}^{p} b_j^2 - \left(\sum_{j=1}^{p} b_j\right)^2} \\ y_a &= \frac{\sum_{j=1}^{p} d_j + T_a \sum_{j=1}^{p} b_j}{p} \end{aligned}\right\} \qquad (17)$$

Thus, T_a and y_a are directly computed from equations (14) and (17). It will be shown later that the values of T_a and y_a obtained using equations (14) and (17) differ very little from the values obtained by the trial-and-error method previously described. This procedure is more fully discussed in the appendix.

Isothermal Data

It has been the general practice to perform stress-rupture tests at constant temperature and to represent these data as shown in figure 4. For this case a least-squares method to obtain the parametric constants, similar to the one previously described, can be used. To do this, the master curves shown in figures 1(b), 2(b), and 3(b) will be presented in the following form:

Larson-Miller parameter:

$$(y + C)(T + 460) = a_0 + a_1x + a_2x^2 + \ldots + a_mx^m$$

Linear parameter:

$$\frac{y - y_a}{T - T_a} = a_0 + a_1x + a_2x^2 + \ldots + a_mx^m \qquad (18)$$

Dorn parameter:

$$y - \frac{D}{T + 460} = a_0 + a_1x + a_2x^2 + \ldots + a_mx^m$$

where $x \equiv \log \sigma$, m is the degree of the polynomial assumed, and y has previously been defined. Note that the coefficients a_i do not, of course, have the same values in the three equations (18).

In equations (18), the master curves are represented by polynomials in $\log \sigma$. In general, a parabola or cubic will describe the master curve with sufficient accuracy except for those materials whose master curve has a reversal in curvature near the tail end at low stresses. Generally, it is desirable to assume a parabola or a cubic first to determine the constants as described herein. The master curve should then be plotted using these constants. If it appears that a reversal in curvature is present at low stresses, the calculation should be repeated omitting these data. Omission of these data should not affect the true value of the parametric constants since these are determinable from any segment of the master curve. It might be expected that increasing the degree of the polynomial will improve the values of the constants. However, it will be shown that polynomials higher than the third degree are generally unnecessary.

The least-squares method will now be applied to equations (18). The quantity that will be minimized is the sum of the squares of the differences between the logarithm of the actual rupture times and the theoretical rupture times, since the rupture time is generally the critical variable.

Larson-Miller parameter. - The first of equations (18) is written as follows:

$$y = -C + a_0\tau + a_1\tau x + a_2\tau x^2 + \ldots + a_m\tau x^m \tag{19}$$

where $\tau = 1/(T + 460)$.

The sum of the squares of the residuals is

$$S = \sum_{i=1}^{n} \left(y_i + C - a_0\tau_i - a_1\tau_i x_i - a_2\tau_i x_i^2 - \ldots - a_m\tau x^m\right)^2 \tag{20}$$

Minimizing the sum of the squares of the residuals as previously done leads to the following set of equations for C and the a_j:

$$\left.\begin{array}{l}
-nC + E_0a_0 + E_1a_1 + \ldots + E_ma_m = \sum_{i=1}^{n} y_i \\
-E_0C + F_0a_0 + F_1a_1 + \ldots + F_ma_m = G_0 \\
-E_1C + F_1a_0 + F_2a_1 + \ldots + F_{m+1}a_m = G_1 \\
\quad\vdots \\
-E_mC + F_ma_0 + F_{m+1}a_1 + \ldots + F_{2m}a_m = G_m
\end{array}\right\} \tag{21}$$

where

$$\left.\begin{aligned} E_j &= \sum_{i=1}^{n} \tau_i x_i^j \\ F_j &= \sum_{i=1}^{n} \tau_i^2 x_i^j \\ G_j &= \sum_{i=1}^{n} y_i \tau_i x_i^j \end{aligned}\right\} \tag{22}$$

The solution of equations (21) gives the optimum value of C as well as the a_j for an assumed degree polynomial for the master curve.

A word of caution is needed with regard to the solution of equations (21). These may in many cases be somewhat ill-conditioned, that is, a large number of significant figures may be lost during the process of solution. Thus, care must be exercised that enough significant figures be carried in the calculation to ensure meaningful answers. The situation is more aggravated the higher the degree of the polynomial assumed for the master curve. However, as will be shown, it generally would not be necessary to assume more than a cubic for the master curve. Note that the number of equations to be solved is equal to the degree of the polynomial assumed plus two.

Linear parameter. - The second of equations (18) is written as follows:

$$y = y_a + (T - T_a)a_0 + (T - T_a)a_1 x + (T - T_a)a_2 x^2 + \ldots + (T - T_a)a_m x^m \tag{23}$$

Equation (23) is of the same form as equation (19) with $-C$ replaced by y_a and τ replaced by $T - T_a$. The solution, therefore, is given by equations (21) and (22) provided T_a is known. A trial-and-error procedure is thus followed as described for the constant-stress data. Values are chosen for T_a, and for each value equations (21) and (22) are solved and S is computed from equation (20) (by replacing C by $-y_a$ and τ by $T - T_a$). The value of T_a for which S is a minimum is the correct value.

Dorn parameter. - The last of equations (18) is written as follows:

$$y = D\tau + a_0 + a_1x + a_2x^2 + \ . \ . \ . + a_mx^m \tag{24}$$

where $\tau = 1/(T + 460)$.

Minimizing the sum of the squares of the residuals leads to the following equations:

$$\left.\begin{aligned}
na_0 + E_0D + H_1a_1 + H_2a_2 + \ . \ . \ . + H_ma_m &= \sum_{i=1}^{n} y_i \\
E_0a_0 + F_0D + E_1a_1 + E_2a_2 + \ . \ . \ . + E_ma_m &= G_0 \\
H_1a_0 + E_1D + H_2a_1 + H_3a_2 + \ . \ . \ . + H_{m+1}a_m &= I_1 \\
H_2a_0 + E_2D + H_3a_1 + H_4a_2 + \ . \ . \ . + H_{m+2}a_m &= I_2 \\
&\vdots \\
H_ma_0 + E_mD + H_{m+1}a_1 + H_{m+2}a_2 + \ . \ . \ . + H_{2m}a_m &= I_m
\end{aligned}\right\} \tag{25}$$

where E_j and G_0 have previously been defined and

$$\left.\begin{aligned}
H &\equiv \sum_{i=1}^{n} x_i^j \\
I &\equiv \sum_{i=1}^{n} y_ix_i^j
\end{aligned}\right\} \tag{26}$$

The solution of equations (25) gives the best values of D and the a_j for a given degree polynomial for the master curve.

EXAMPLES

Constant-Nominal-Stress Data

Consider a set of constant-stress data as given in table I and taken from a tabulation of the data in reference 4. It is seen from the table that

$$n_1 = 9, \ n_2 = 9, \ n_3 = 9, \ n_4 = 8, \ n = 35$$

Letting $\tau_i = \frac{1}{T_i + 460}$ (eqs. (4)) gives

$A_1 = 5.1709\times10^{-3}$ $B_1 = 2.9758\times10^{-6}$ $C_1 = 8.0785\times10^{-3}$ $D_1 = 13.748$

$A_2 = 5.2997\times10^{-3}$ $B_2 = 3.1311\times10^{-6}$ $C_2 = 5.6297\times10^{-3}$ $D_2 = 8.8652$

$A_3 = 5.5102\times10^{-3}$ $B_3 = 3.3793\times10^{-6}$ $C_3 = 4.3022\times10^{-3}$ $D_3 = 6.6343$

$A_4 = 5.3898\times10^{-3}$ $B_4 = 3.6358\times10^{-6}$ $C_4 = 5.3599\times10^{-3}$ $D_4 = 7.6130$

and

$$\sum_{i=1}^{35} y_i = D_1 + D_2 + D_3 + I_4 = 36.861$$

Then, from equations (5)

$$C = 23.8$$

$$b_1 = 44{,}146 \quad b_2 = 42{,}154 \quad b_3 = 4C{,}151 \quad b_4 = 36{,}819$$

The best value of C in this case is therefore 23.8.

By using equations (9), the Dorn parameter is calculated as

$$D = 41{,}470$$

To calculate the constants for the linear parameter by the trial-and-error method described, values must be assumed for T_a. If it is not known at all what region T_a might be in, a rough plot could be made to determine a first guess for T_a. Thus, assuming $T_a = 500$ and, in this case, defining $\tau = T - T_a$ (eqs. (4)) give

$A_1 = 7{,}049.99$ $B_1 = 5.56625\times10^6$ $C_1 = 10.2311\times10^3$

$A_2 = 6{,}694.99$ $B_2 = 5.06842\times10^6$ $C_2 = 5.39997\times10^3$

$A_3 = 6{,}084.98$ $B_3 = 4.15518\times10^6$ $C_3 = 3.84065\times10^3$

$A_4 = 4{,}209.98$ $B_4 = 2.23945\times10^6$ $C_4 = 3.48307\times10^3$

and, as before,

$$\sum_{i=1}^{35} y_i = 36.8608$$

Therefore, from equations (5), by letting y_a take the place of $-C$, there is obtained

$$y_a = 11.4$$

$$b_1 = -0.001266 \quad b_2 = -0.001405 \quad b_3 = -0.001583 \quad b_4 = -0.001996$$

Then, from equation (2),

$$S = 0.6707$$

Now, by assuming a value of T_a equal to 600, the computations are repeated giving

$$y_a = 9.896$$

$$S = 0.5819$$

and, for $T_a = 700$,

$$y_a = 8.306$$

$$S = 0.5889$$

These calculations indicate a minimum value for S at $T_a = 600$. If greater accuracy is desired, a few more points can be taken in this vicinity. Thus, it turns out that the true minimum is approximately at

$$T_a = 650$$

$$y_a = 9.108$$

$$S = 0.5640$$

However, as will be shown later, using values of $T_a = 600$, $y_a = 9.9$ or $T_a = 700$, $y_a = 8.3$ will not affect the stress-rupture results appreciably.

This trial-and-error procedure can be completely avoided if the approximate method described for the linear parameter is used. Thus, by defining $\tau \equiv T$, equations (4) give

$$
\begin{array}{llll}
A_1 = 11{,}550 & B_1 = 14.866\times10^6 & C_1 = 17{,}105 & D_1 = 13.748 \\
A_2 = 11{,}195 & B_2 = 14.013\times10^6 & C_2 = 9{,}832.6 & D_2 = 8.8652 \\
A_3 = 10{,}585 & B_3 = 12.490\times10^6 & C_3 = 7{,}157.8 & D_3 = 6.6343 \\
A_4 = 8{,}210 & B_4 = 8.4494\times10^6 & C_4 = 7{,}289.6 & D_4 = 7.6130
\end{array}
$$

and from equations (14),

$$
\begin{array}{ll}
d_1 = 17.4 & b_1 = -0.0124 \\
d_2 = 17.9 & b_2 = -0.0136 \\
d_3 = 19.3 & b_3 = -0.0158 \\
d_4 = 23.4 & b_4 = -0.0219
\end{array}
$$

Substituting into equations (17) gives

$$T_a = 643$$

$$y_a = 9.3$$

In this case the trial-and-error procedure is not really necessary as good results can be obtained by using equations (14) and (17) to get T_a and y_a.

Isothermal Data

Consider the isothermal data of figure 4 tabulated in table II. To calculate the best parameter C, let $\tau \equiv \frac{1}{T_j + 460}$ and assume a third-degree polynomial ($m = 3$) for the master curve. Then computing the E_j, F_j, and G_j by equations (22) and substituting into equations (21), with

$m = 3$, give the following five equations:

$$-32C + 0.017013000\ a_0 + 0.067135551\ a_1 + 0.26671741\ a_2 +$$
$$1.0666639\ a_3 = 62.095000$$

$$-0.017013000\ C + 9.1227245\times10^{-6}\ a_0 + 36.213995\times10^{-6}\ a_1 +$$
$$144.71013\times10^{-6}\ a_2 + 582.01515\times10^{-6}\ a_3 = 0.033195744$$

$$-0.067135551\ C + 36.213995\times10^{-6}\ a_0 + 144.71013\times10^{-6}\ a_1 +$$
$$582.01515\times10^{-6}\ a_2 + 2355.6764\times10^{-6}\ a_3 = 0.12753108$$

$$-0.26671714\ C + 144.71013\times10^{-6}\ a_0 + 582.01515\times10^{-6}\ a_1 +$$
$$2355.6764\times10^{-6}\ a_2 + 9593.2956\times10^{-6}\ a_3 = 0.49259087$$

$$-1.0666639\ C + 582.01515\times10^{-6}\ a_0 + 2355.6764\times10^{-6}\ a_1 +$$
$$9593.2956\times10^{-6}\ a_2 + 0.039301429\ a_3 = 1.9127579$$

The solution of these equations gives

$$C = 15.3$$

$$a_0 = 182{,}925 \quad a_1 = -92.231.9 \quad a_2 = 20{,}404.8 \quad a_3 = -1{,}697.03$$

To calculate the linear parameter constants T_a, y_a, let $\tau_i = T_i - T_a$ and replace C by $-y_a$. For an assumed value of T_a, E_j, F_j, and G_j are computed as before from equations (22). Thus, with $m = 3$ and $T_a = 0$, equations (21) become

$$32y_a + 46{,}000\ a_0 + 178{,}945.27\ a_1 + 700{,}948.43\ a_2 + 2{,}764{,}570.3\ a_3 = 62.095$$

$$46{,}000\ y_a + 67.16\times10^6\ a_0 + 259.11478\times10^6\ a_1 + 1006.6607\times10^6\ a_2 +$$
$$3937.9381\times10^6\ a_3 = 88{,}481$$

$$178{,}945.27\ y_a + 259.11478\times10^6\ a_0 + 1006.6607\times10^6\ a_1 + 3937.9381\times10^6\ a_2 +$$
$$15{,}510.467\times10^6\ a_3 = 335{,}601.61$$

$$700{,}948.43\ y_a + 1006.6607\times10^6\ a_0 + 3937.9381\times10^6\ a_1 + 15{,}510.467\times10^6\ a_2 +$$

$$61{,}505.824\times10^6\ a_3 = 1{,}280{,}135.8$$

$$2{,}764{,}570.3\ y_a + 3937.9381\times10^6\ a_0 + 15{,}510.467\times10^6\ a_1 + 61{,}505.824\times10^6\ a_2 +$$

$$245{,}526.23\ a_3 = 4{,}910{,}487.4$$

The solution of these equations gives a value of $y_a = 15.0$. The value of S obtained from equation (20) is 0.4731. Choosing a value of T_a equal to 100 and repeating the previous calculations result in a value of $y_a = 14.1$ and $S = 0.4801$. A minimum value of $S = 0.4671$ is obtained for $T_a = -300$ with a corresponding $y_a = 17.7$. A plot of S against T_a and the corresponding values of y_a is shown in figure 5. It is seen that S does not change much with T_a in the range of 0 to -500. It would, therefore, not make much difference which value of T_a in this range is used as long as the corresponding value of y_a is used with it. For other materials, the curve of S plotted against T_a might have a sharper minimum, and the value of T_a would be more critical.

For the Dorn parameter, E_j and G_0 are computed as for the Larson-Miller parameter, and H_j and I_j are computed using equations (26) $\left(\text{with } \tau_i = \frac{1}{T_i + 460}\right)$. Then, for $m = 3$, the following equations are obtained:

$$32a_0 + 0.017013000\ D + 125.50874\ a_1 + 495.64303\ a_2 +$$

$$1970.5813\ a_3 = 62.095000$$

$$0.017013000\ a_0 + 0.0000091227245\ D + 0.067135551\ a_1 + 0.26671741\ a_2 +$$

$$1.0666639\ a_3 = 0.033195744$$

$$125.50874\ a_0 + 0.067135551\ D + 495.64303\ a_1 + 1970.5813\ a_2 +$$

$$7886.7697\ a_3 = 237.26461$$

$$495.64303\ a_0 + 0.26671741\ D + 1970.5813\ a_1 + 7886.7696\ a_2 +$$

$$31{,}770.400\ a_3 = 911.60106$$

$$1970.5813\ a_0 + 1.0666639\ D + 7886.7697\ a_1 + 31{,}770.400\ a_2 +$$

$$128{,}793.83\ a_3 = 3521.6275$$

Solving these equations gives

$$D = 32{,}900$$

RESULTS AND DISCUSSION

Determination of Master Curves

In order to determine how well the various parameters are determined by the least-squares method presented, master curves for the linear parameter were computed for the examples described. These master curves are plotted for the constant-stress data of the 17-22A(S) steel and the isothermal data of the 18-8 stainless in figures 6 and 7, respectively. These curves were then used to replot the constant-stress lines for the 17-22A(S) and the isothermal curves for the 18-8 stainless. The results in figures 8 and 9 show that the agreement between experimental data and the computed lines is good.

Effect of Polynomial Approximation

As a further check on the assumption that the master curves can be represented by polynomials as given in equations (18), the parameters for the constant-stress data of the 17-22A(S) were recomputed using a polynomial representation for the master curve. Thus, equations (21) were solved for the linear parameters assuming polynomials of the third degree and also of the fifth degree for the master curves. The values obtained for both the third-degree and the fifth-degree polynomials were 23.8 for C, 650 for T_a, and 9.1 for $\log t_a$. These values are seen to be the same as previously obtained using the constant-stress data directly to find the best intersection of the straight lines.

As a further illustration, the data for 25-20 steel given in reference 2 were considered. The parameters for this material were obtained using second-, third-, fourth-, and fifth-degree polynomials. The following results are obtained:

$$m = 2,\ C = 14.4,\ T_a = -400,\ \log t_a = 17.3$$

$$m = 3,\ C = 14.2,\ T_a = -200,\ \log t_a = 15.3$$

$$m = 4,\ C = 14.4,\ T_a = -200,\ \log t_a = 15.3$$

$$m = 5,\ C = 14.6,\ T_a = -200,\ \log t_a = 15.6$$

It is seen that the same results were obtained for all the polynomials except for the polynomial of second degree which gave slightly different values for the linear parameter constants. However, there is very little difference in the sum of the squares of the deviations in going from $T_a = -200$ to $T_a = -400$ so that even using a value of $m = 2$ would give good results in this case.

Insensitivity of Results to Parameter Values

As a further illustration of the relative insensitivity of creep-rupture-data correlation to the precise values of the linear parameter constants (as long as T_a and $\log t_a$ appear in the proper combination), the data for Nimonic 80A as given in table III were analyzed. These data were taken from reference 5 where the parametric constants are given as 16.9 for C, 660° F for T_a, and 9.65 for $\log t_a$. In a private communication to the authors of the present paper other investigators questioned the values given in reference 5 stating that their analysis using the same data gave values of $T_a = 100°$ F, $\log t_a = 1.6$.

An analysis was therefore made of these data using the least-squares method presented herein. The results are shown in figures 10 and 11. Figure 10 shows that the best values for the parameters are $T_a = 400°$ F, $\log t_a = 12.2$. However, because the curve is flat in the region of its minimum (it is drawn here to a very expanded scale in order to show the precise minimum), other combinations of constants show sums of deviations not much higher than those at the minimum. Thus, the sum of the squares of the deviations S for the minimum point is in the neighborhood of 5.5×10^{-2}, the value for the constants of reference 5 is approximately 6×10^{-2}, and for the constants of the private communication it is approximately 5.7×10^{-2}. Thus, all three combinations are for all practical purposes equally good. This is further illustrated in figure 11 where the reconstructed isothermals for each of the three combinations are compared to the experimental data. All the computed values lie on the same solid lines. It should be noted that all three combinations lie on the straight-line plot of $\log t_a$ against T_a of figure 10.

This illustration shows graphically how different investigators analyzing the same data can arrive at different values of the constants in the linear parameter unless an objective method such as the least-squares method presented herein is used. It also shows, however, that the degree of correlation of the data is rather insensitive to the precise values of the parameters as long as the proper value of $\log t_a$ is used with T_a. This insensitivity of the correlation to the precise values of the constants is due to the fact that the intersection point of the constant-stress lines is generally remote from the actual data points. Therefore, moving the intersection point along an average line through all the data would not appreciably change the individual lines. It has been the experience of the authors that this is true for most materials.

CONCLUSIONS

An objective least-squares method has been presented for determining the optimum values of creep-rupture parametric constants for the Larson-Miller, linear, and Dorn parameters. From the examples shown it is concluded that the results obtained are insensitive to the degree of polynomial assumed for the master curve and that for the linear parameter the actual values of T_a and $\log t_a$ are not critical as long as they appear in the proper combination. Furthermore, the method permits a person with no experience in the field of materials to obtain the correct values of the parametric constants from tabular data.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, December 12, 1958

APPENDIX - APPROXIMATE METHOD FOR CONSTANT-STRESS DATA

Consider a set of constant-stress data as shown in figure 8. The trial-and-error method for obtaining the best intersection point for these lines utilizes the least-squares procedure by minimizing simultaneously the squares of the deviation of all the data points from the lines. By this method each point is given the same weight.

The approximate method described in the body of this report first treats each set of constant-stress data separately and finds the best fitting straight line for the set. Now, if the set of straight lines thus determined is to intersect at a common point, a plot of slope against intercept for these lines should be a straight line as can be seen from equations (11) where b_j is the slope of the j^{th} line and d_j is the intercept. The so-called "best average" intersection point is therefore found by fitting the best straight line to the plot of d_j against b_j.

It is to be noted that by this approximate procedure each constant-stress line is given the same weight without regard to the number of data points associated with that stress. If the data are such that one or more of the lines is ill defined then it may be desirable to employ some weighting procedure when using the approximate method.

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TABLE I. - STRESS-RUPTURE DATA FOR

17-22A(S) (REF. 4)

σ	T	t	σ	T	t
10,000	1370	2.8	40,000	1285	0.075
	1370	3.7		1260	.37
	1350	4.5		1210	1.35
	1315	12.5		1210	1.90
	1270	48.5		1175	6.60
	1270	51.3		1150	13.6
	1235	129.8		1120	39.5
	1210	228.7		1100	83.0
	1160	1301		1075	205.7
20,000	1400	0.1	80,000	1140	0.033
	1375	.12		1070	1.5
	1320	1.0		1045	2.5
	1270	3.9		1030	5.7
	1230	13.3		1000	15
	1190	48.0		985	82
	1170	102.7		970	109.2
	1140	242.1		970	433
	1100	987			

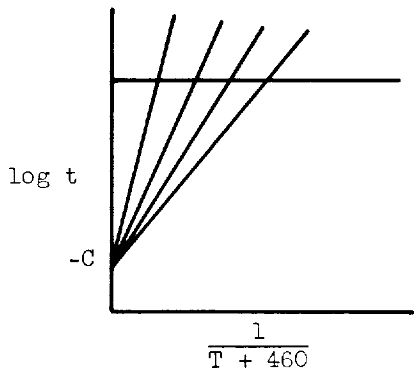
TABLE II. - 18-8 STAINLESS STEEL (REF. 2)

Temperature, T	log t	σ	log σ	Temperature, T	log t	σ	log σ
1200	0	38.0×10^3	4.579784	1600	0.55	9.0×10^3	3.95424
	.575	32.0	4.505150		.90	8.0	3.90309
	1.04	28.0	4.447158				
	2.48	18.0	4.255272		2.39	4.0	3.60206
	3.00	14.0	4.146128		2.66	3.5	3.544068
	3.49	11.5	4.060698		3.00	3.0	3.477121
					3.26	2.76	3.440909
1300	0.55	22.0×10^3	4.342423	1800	0.08	5.0×10^3	3.69897
	1.16	18.5	4.267172		.91	3.55	3.55022
	2.28	12.5	4.096910		2.68	1.65	3.217484
	2.79	10.0	4.0				
	3.30	8.0	3.903090				
	3.90	6.0	3.778151				
1400	0.40	18.0×10^3	4.255272				
	.81	14.6	4.164353				
	2.86	7.0	3.845098				
	3.08	6.0	3.778151				
	3.52	5.0	3.698970				
1500	0.66	11.0×10^3	4.04393				
	.77	9.4	3.973128				
	1.41	8.0	3.903090				
	2.12	6.0	3.778151				
	2.45	5.0	3.698970				
	3.02	4.0	3.602060				

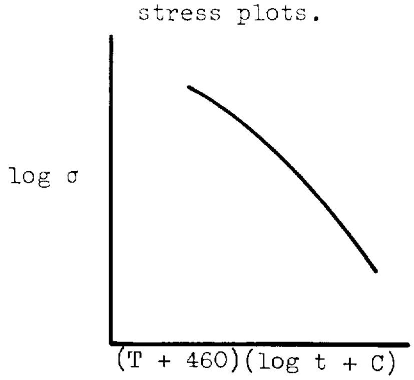
TABLE III. - RUPTURE DATA FROM NIMONIC 80A

(FROM REF. 5)

650° C		700° C		750° C	
Stress, tons/sq in.	Life, hr	Stress, tons/sq in.	Life, hr	Stress, tons/sq in.	Life, hr
30	274	23	208	17	138
28	481	21	443	16	230
26	898	19	683	14	419
24	1,292	16	1,735	12	852
22	2,655	13	4,836	10	1,857
20	5,270	10	10,896	8	4,450
18	8,171	7	34,053	6	13,089
16	13,386			4	22,657

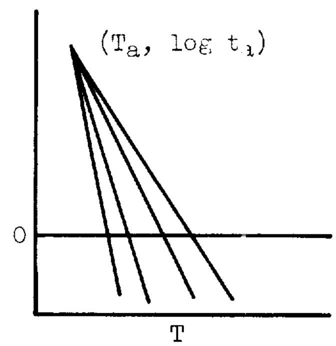


(a) Constant-nominal-stress plots.

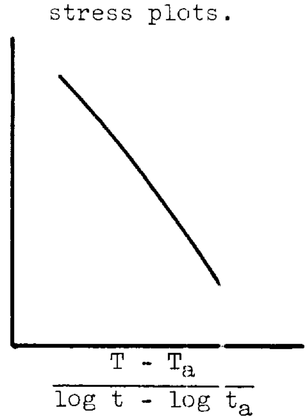


(b) Master curve.

Figure 1. - Larson-Miller parameter method for extrapolation (ref. 1).

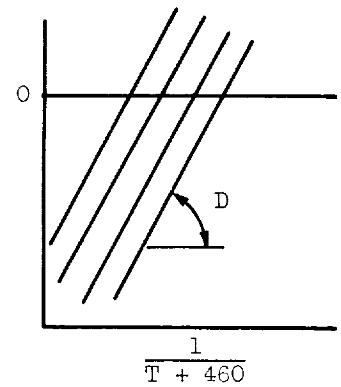


(a) Constant-nominal-stress plots.

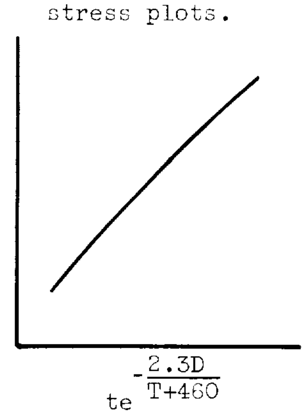


(b) Master curve.

Figure 2. - Linear parameter method for extrapolation (ref. 2).



(a) Constant-nominal-stress plots.



(b) Master curve.

Figure 3. - Dorn parameter method for extrapolation (ref. 3).

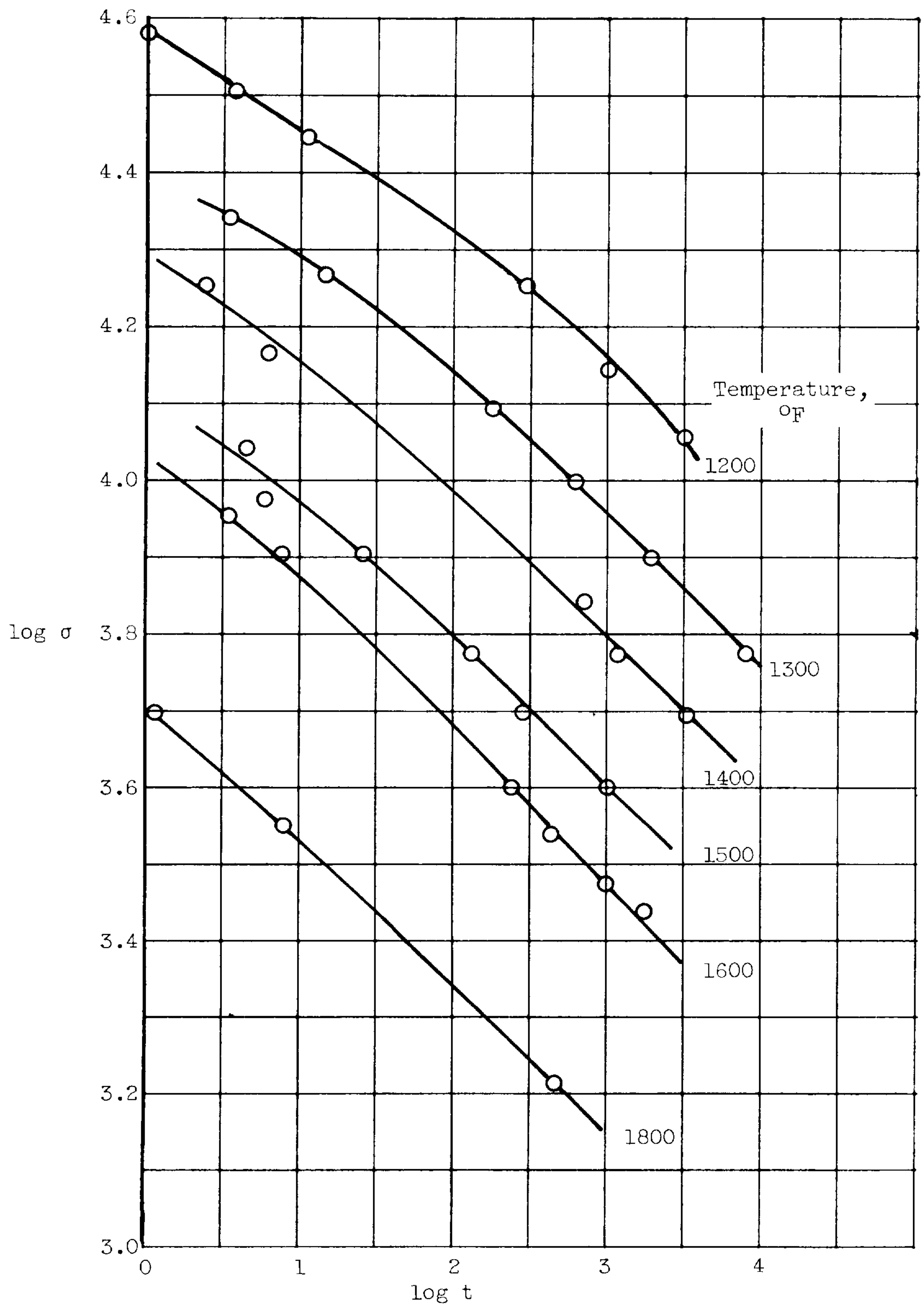


Figure 4. - Isothermal stress-rupture data for 18-8 stainless steel (ref. 2).

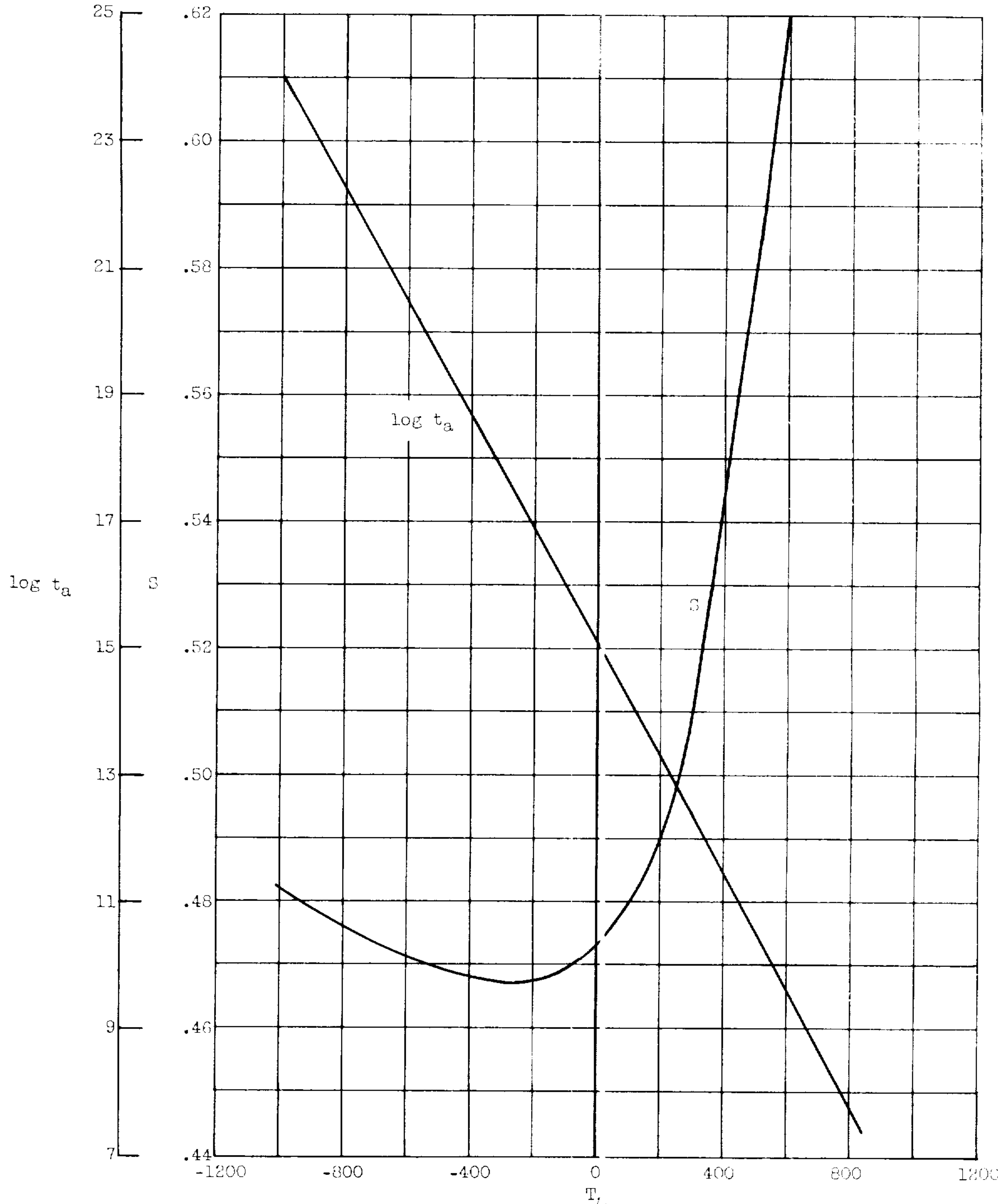


Figure 5. - Variation of S and log t_a with T_a for 18-8 stainless.

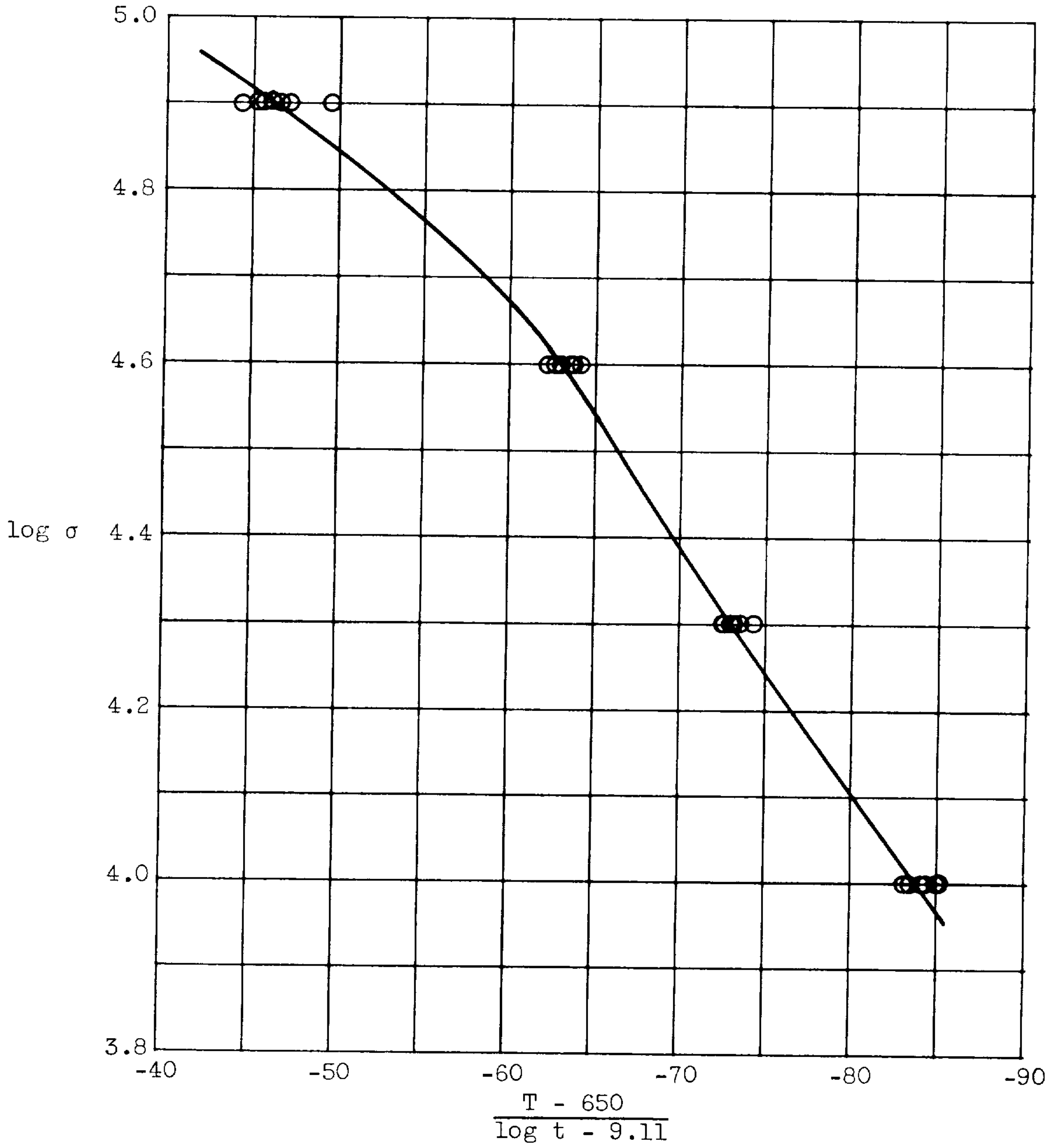


Figure 6. - Master curve for 17-22A(S).

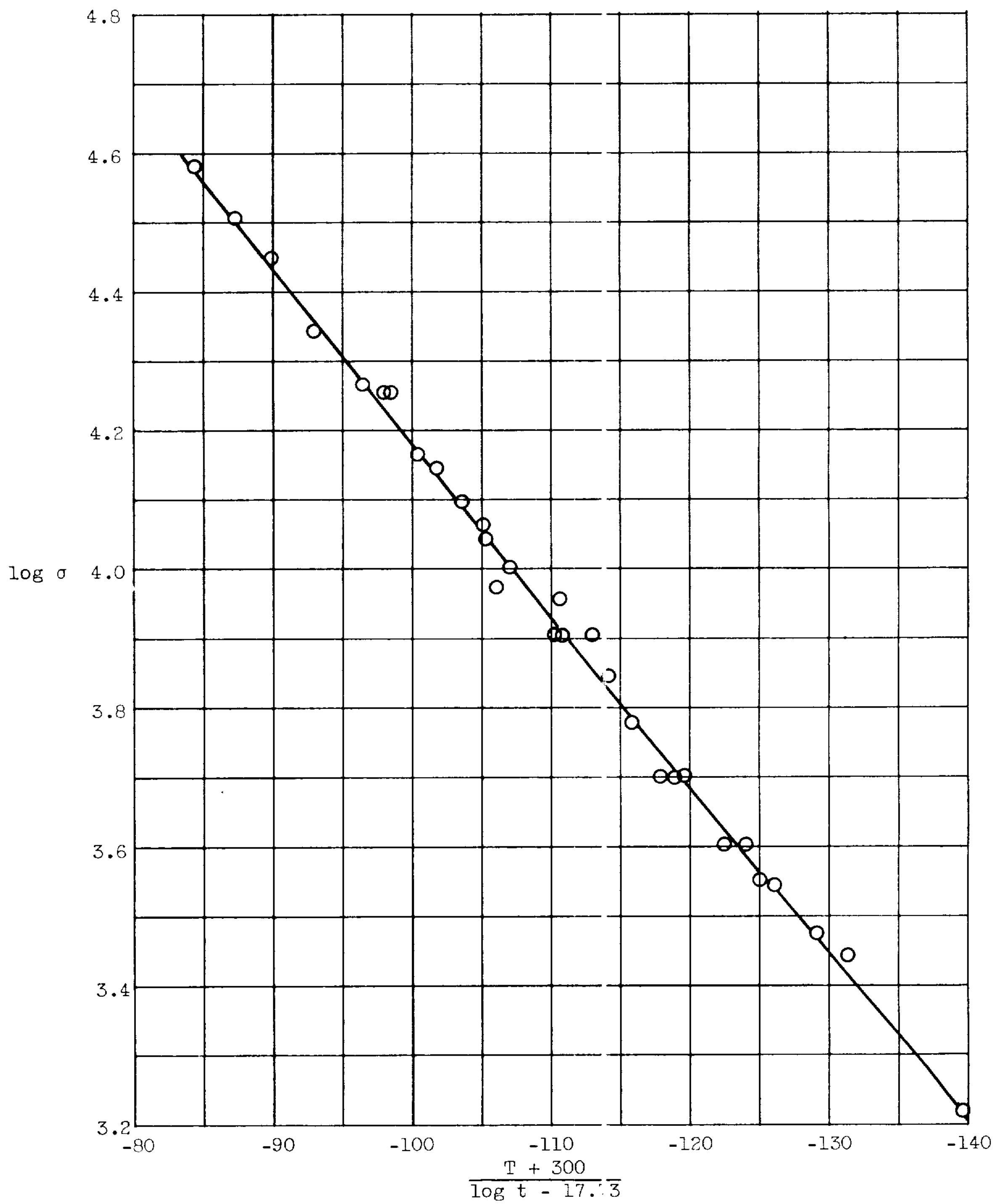


Figure 7. - Master curve for 18-8 stainless.

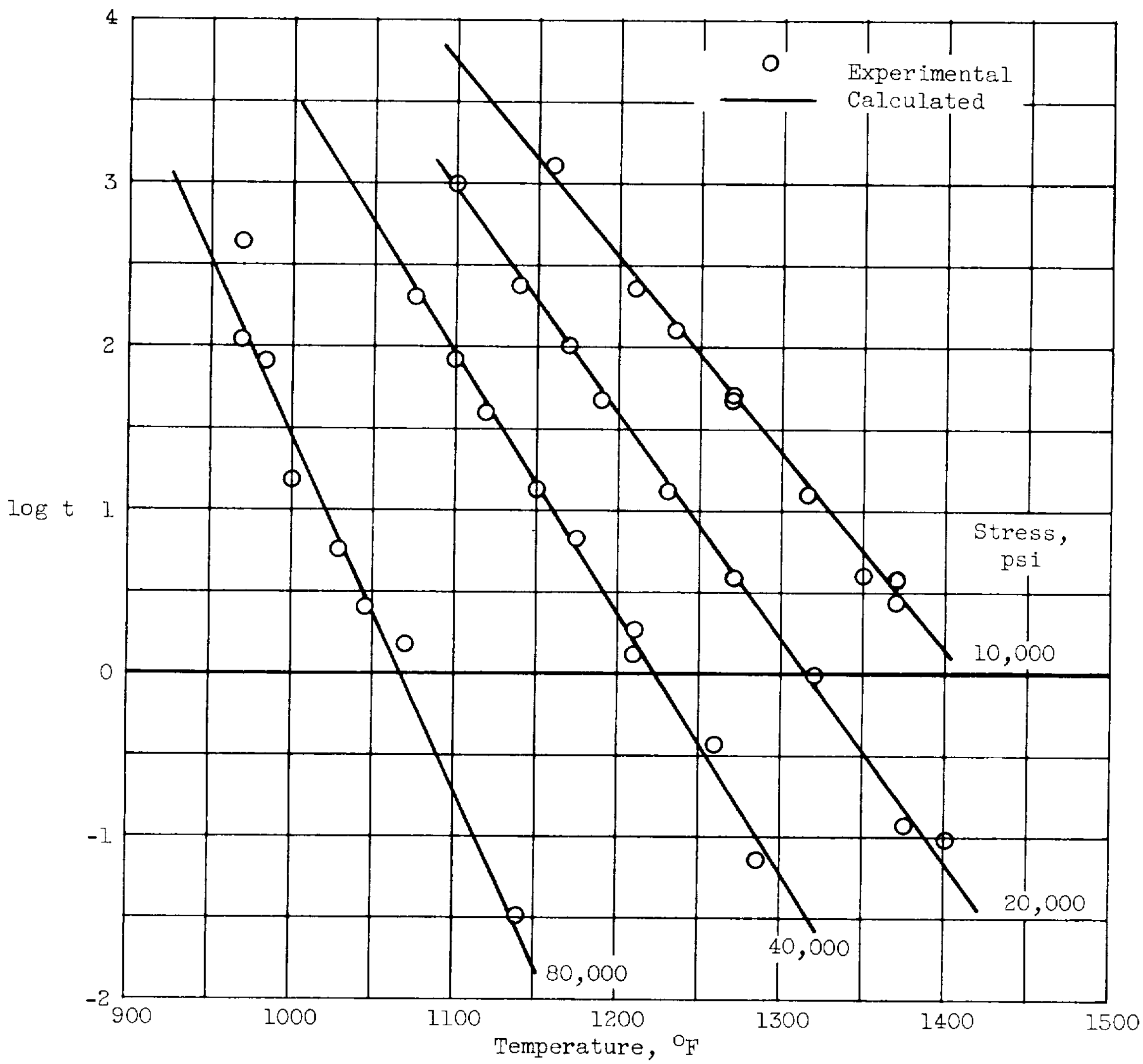


Figure 8. - Comparison of computed isostress lines with experimental data for 17-22A(S).

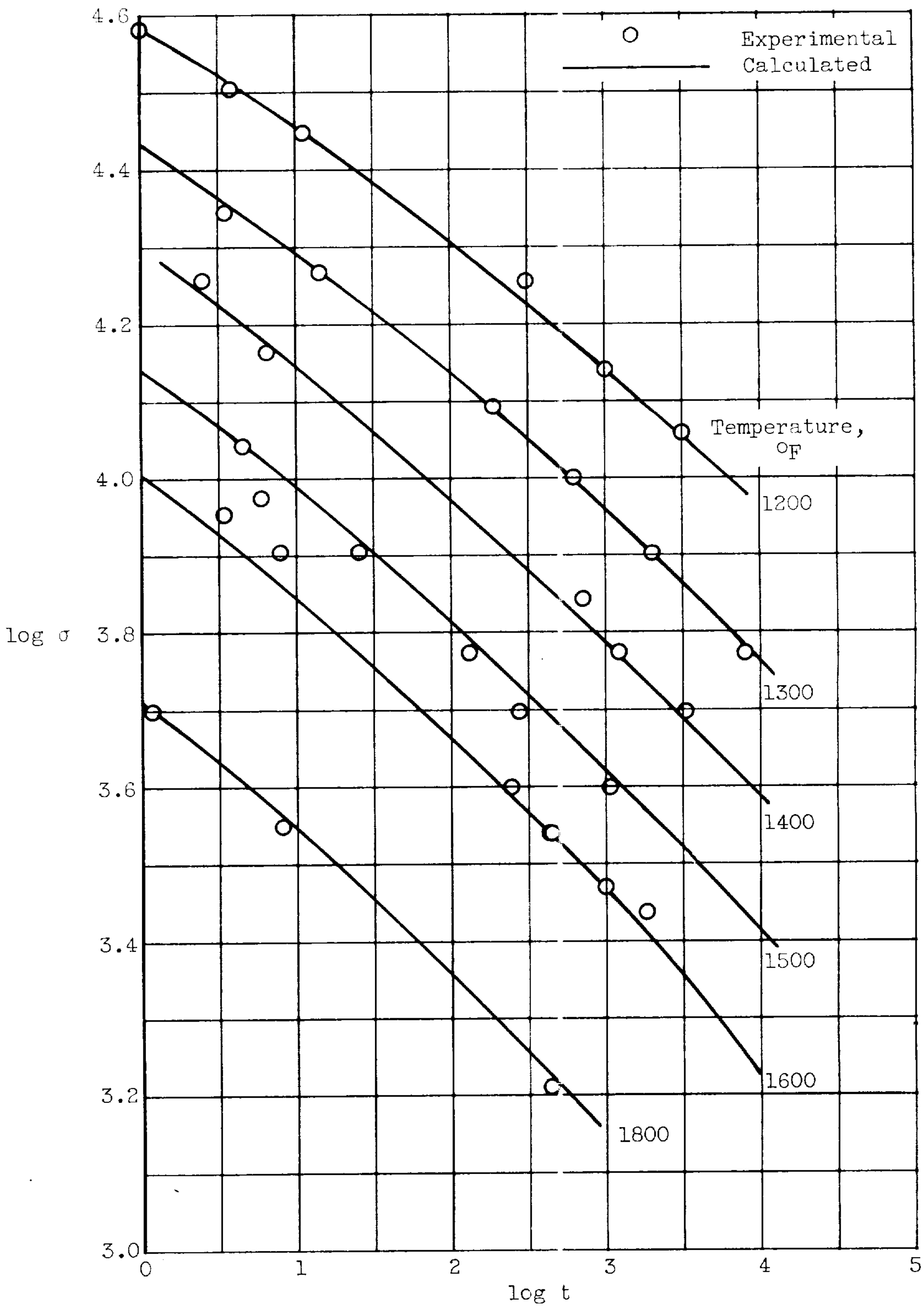


Figure 9. - Comparison of calculated isothermals with experimental data for 18-8 stainless.

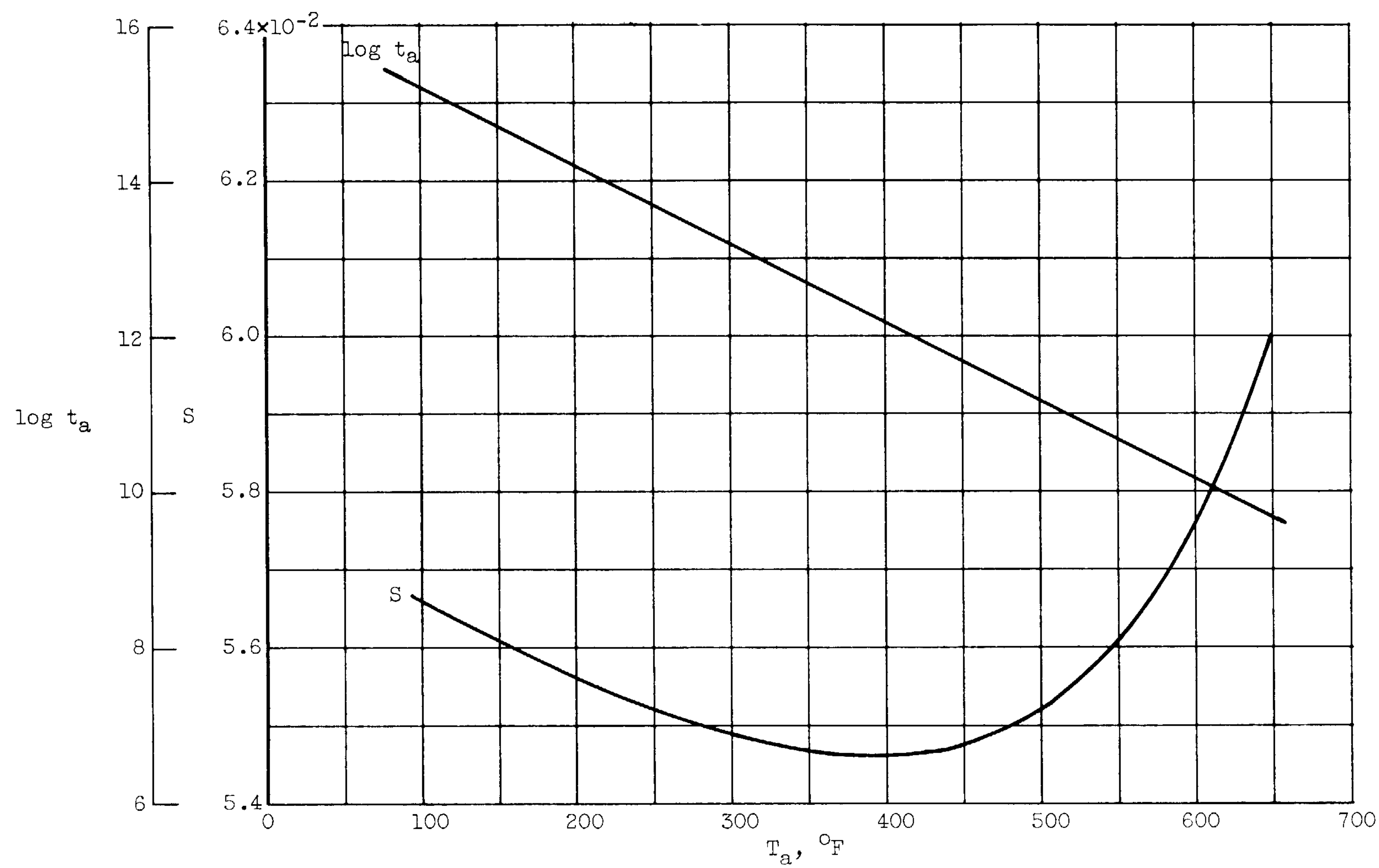


Figure 10. - Evaluation of best values for T_a and log t_a for Nimonic 80A using data from reference 5.

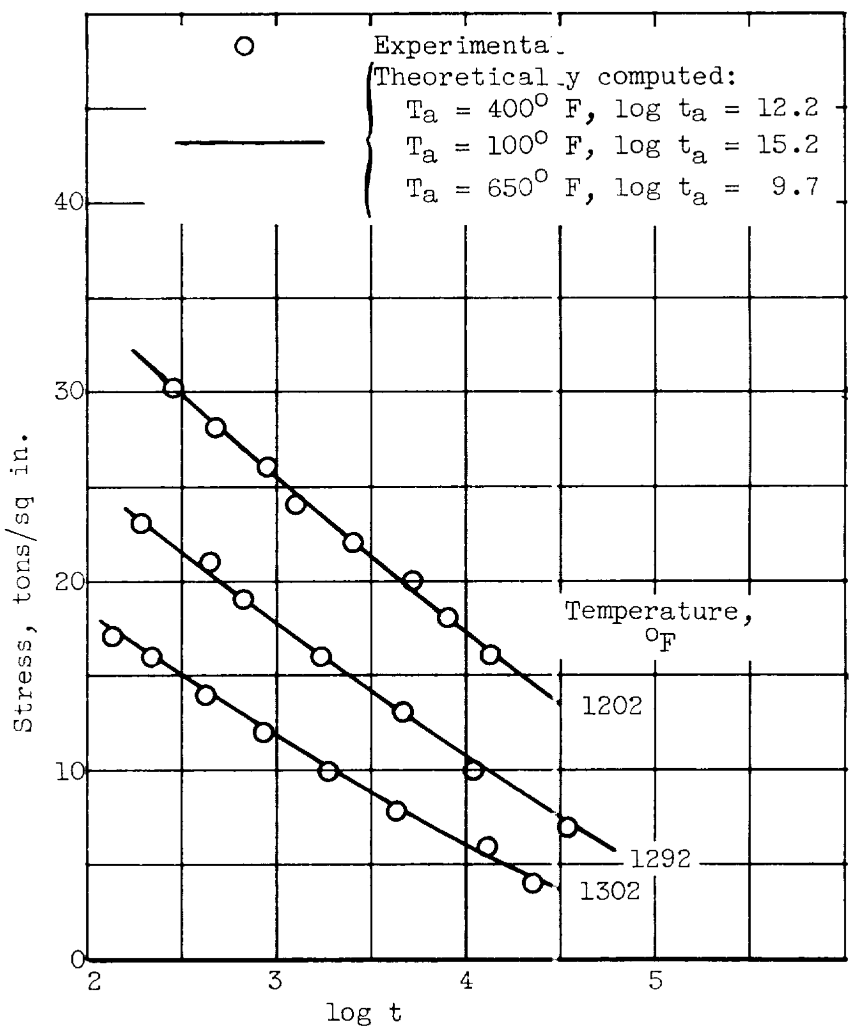


Figure 11. - Isothermal curves for Nimonic 80A.